Bab 4 : Aplikasi Integral Tertentu

4.1 Luas Antara Dua Kurva

Diberikan fungsi kontinu f dan g pada $[a, b]$ dengan:

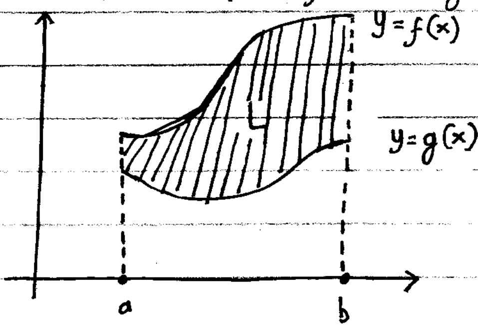
$f(x) \geq g(x)$ untuk $a \leq x \leq b$

$\downarrow$

Kurva $f(x)$ di atas kurva $g(x)$

$$L = [L.\text{bawah } f(x)] - [L.\text{bawah } g(x)]$$
$$= \int_a^b f(x)\,dx - \int_a^b g(x)\,dx$$
$$= \int_a^b [f(x) - g(x)]\,dx$$
$$\boxed{L = \int_a^b [f(x) - g(x)]\,dx}$$

Berikut ini ilustrasi kurva $f(x)$ dan $g(x)$

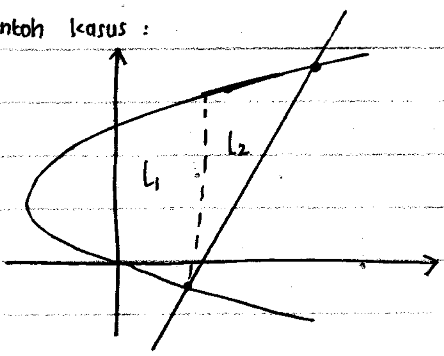


Luas Antara $x = v(y)$ dan $x = w(y)$

Selain mengintegralkan dalam fungsi x, dapat juga menentukan luas daerah dengan mengintegralkan dalam fungsi y.

※ Kenapa harus dalam fungsi y?

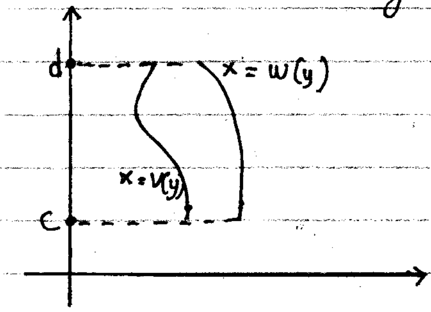
Contoh kasus :



-> Apabila mencari luas terhadap x, harus dibagi menjadi 2 luasan L_1 dan L_2

-> Apabila terhadap $y \longrightarrow$ hanya integral 1 kali.

※ Rumus luas terhadap y



Jika w dan v fungsi kontinu dan jika $w(y) \geq v(y)$ untuk semua y di $[c, d]$

$\downarrow$

w berada di kanan
v berada di kiri

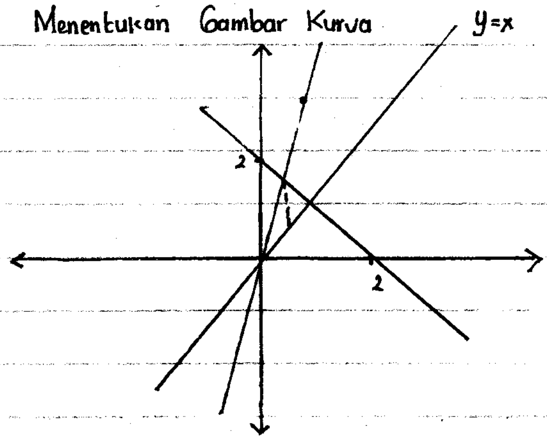
$$\boxed{L = \int_c^d [w(y) - v(y)]\,dy}$$

※ Contoh soal !

Dapatkan luas daerah yang dibatasi kurva :

$y = x$; $y = 4x$; $y = -x + 2$

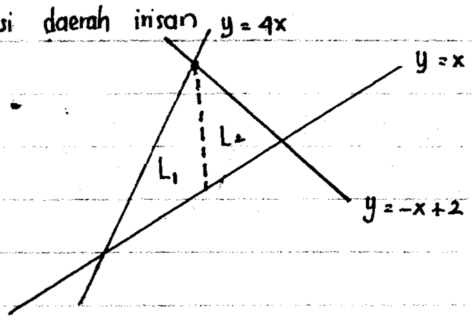
① Menentukan Gambar Kurva



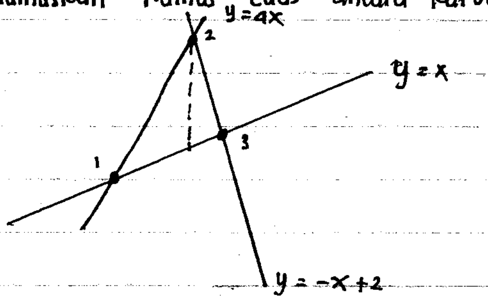
*) cara menggambar

- $y = x$ $(1,1)(0,0)$
- $y = 4x$ $(0,0)(1,4)$
- $y = -x + 2$ $(0,2)(2,0)$

Ilustrasi daerah irisan



(2) Merumuskan Rumus Luas antara kurva



-> Titik potong 1

$$y_1 = y_2$$
$$x = 4x$$
$$3x = 0$$
$$x = 0 \longrightarrow y = 0 \quad (0,0)$$

-> Titik potong 2

$$y_1 = y_2$$
$$4x = -x + 2$$
$$5x = 2$$
$$x = \frac{2}{5}$$

-> Titik potong 3

$$y_1 = y_2$$
$$x = -x + 2$$
$$2x = 2$$
$$x = 1$$

(3) Menuliskan rumus

$$L = L_1 + L_2$$
$$= \int_0^{\frac{2}{5}} (4x - x)\,dx + \int_{2/5}^{1} (-x + 2 - x)\,dx$$
$$= \int_0^{\frac{2}{5}} 3x\,dx + \int_{2/5}^{1} (2 - 2x)\,dx$$
$$= \frac{3}{2}x^2 \Big]_0^{2/5} + 2x - x^2 \Big]_{2/5}^{1}$$
$$= \left[\frac{3}{2}\left(\frac{2}{5}\right)^2 - \frac{3}{2}(0)^2\right] + \left[2(1) - (1)^2 - \left(2\left(\frac{2}{5}\right) - \left(\frac{2}{5}\right)^2\right)\right]$$
$$= \left[\frac{3}{\cancel{2}} \cdot \frac{\cancel{4}^{2}}{25}\right] + \left[2 - 1 - \frac{4}{5} + \frac{4}{25}\right]$$
$$= \left[\frac{6}{25}\right] + \left[1 - \frac{16}{25}\right]$$
$$= 1 - \frac{10}{25}$$
$$= \frac{15}{25} \rightsquigarrow \frac{3}{5} \text{ satuan luas}$$

※ Latihan Soal!

Dapatkan luas yang dibatasi kurva

a). $y = x^2$; $y = \sqrt{x}$; $x = \frac{1}{4}$; $x = 1$

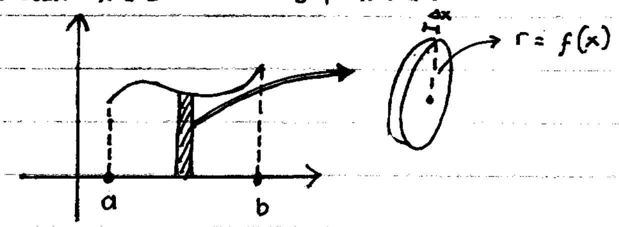
b). $y = x^2$; sumbu-y ; $y = 1$; $y = 4$

▶ 4.2 Volume Benda Putar

Metode - Metode dalam Menentukan Volume Benda Putar

(1) Metode Cakram

Misalkan terdapat R daerah yang batas atasnya adalah grafik f, batas bawah sb. x, dan sisi-sisinya dibatasi $x = a$ dan $x = b$, maka :



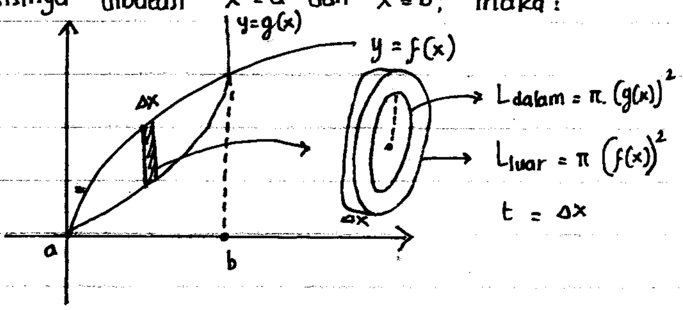
Apabila diambil partisi ⊥ sumbu putar :

$$V_{tabung} = La \times t$$
$$= \pi . r^2 \times t$$
$$= \pi . [f(x)]^2 . \Delta x$$
$$= \int_a^b \pi [f(x)]^2\,dx$$

$$\boxed{V_{cakram} = \int_a^b \pi . [f(x)]^2\,dx}$$

(2) Metode Cincin

Misalkan terdapat R daerah yang batas atasnya adalah grafik f, batas bawah grafik g, dan sisi-sisinya dibatasi $x = a$ dan $x = b$, maka :



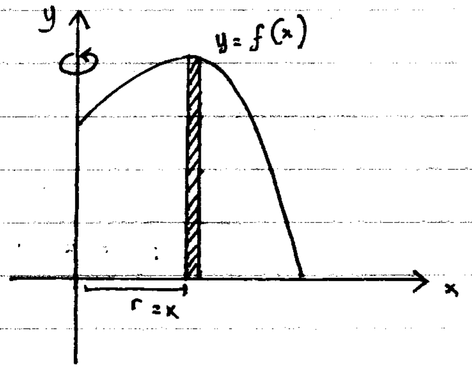
$$V_{total} = V_{luar} - V_{dalam}$$
$$= \pi . r_{luar}^2 . t - \pi . r_{dalam}^2 . t$$
$$= \pi . t \left[(f(x))^2 - (g(x))^2\right].$$
$$V = \int_a^b \pi \left[(f(x))^2 - (g(x))^2\right] dx$$

$$\boxed{V_{cincin} = \int^b \pi \left[(f(x))^2 - (g(x))^2\right] dx}$$

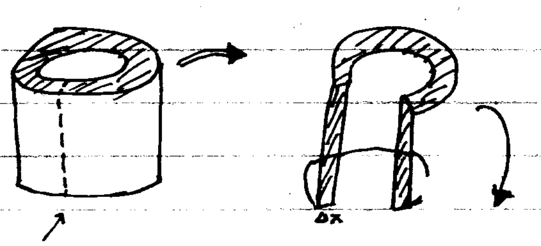
③. Metode Kulit Tabung

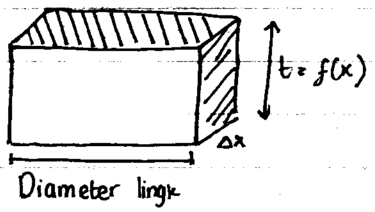
Misalkan terdapat benda padat dibatasi dua silinder tegak dengan pusat yang sama.

※ ~~Konsep : Partisi diambil sejajar dengan sumbu putar~~



Apabila hanya memutar daerah partisi, maka diperoleh :





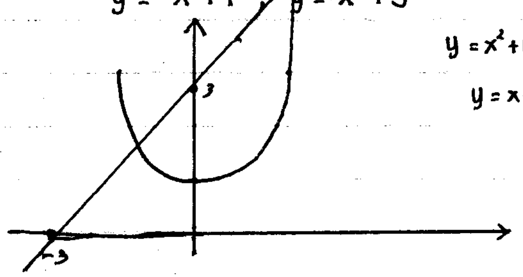
$$V_{balok} = p \times l \times t$$
$$= 2\pi . r . \Delta x . t$$
$$= 2\pi r t . \Delta x$$
$$V_{kulit} = \int 2\pi . x (f(x)) . dx$$

$$\boxed{V_{kulit\ tabung} = \int_a^b 2\pi x [f(x)]\, dx}$$

Contoh Soal

Dapatkan volume benda padat dari kurva yang dibatasi grafik berikut apabila diputar thd sumbu x.

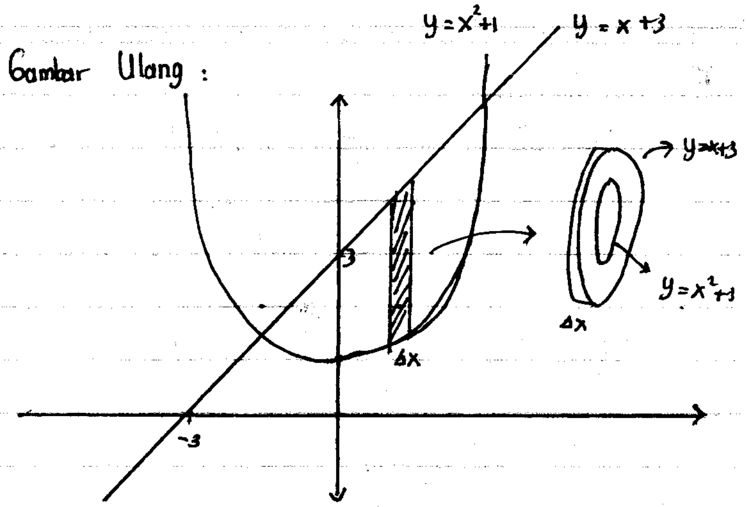
$y = x^2 + 1$; $y = x + 3$



$y = x^2 + 1 \rightarrow (0,1)\ (\pm 1, 0)$

$y = x + 3 \rightarrow (0,3)\ (-3,0)$

Gambar Ulang :



$$V = V_{luar} - V_{dalam}$$
$$= \int_a^b \pi [x+3]^2 . dx - \int_a^b \pi [x^2+1]^2 dx$$

→ Menentukan a dan b

$$y_1 = y_2 \longrightarrow x^2 + 1 = x + 3$$
$$x^2 - x - 2 = 0$$
$$(x-2)(x+1) = 0$$
$$x = 2 \vee x = -1$$

$$V_{cincin} = \int_{-1}^{2} \pi \left[(x+3)^2 - (x^2+1)^2 \right] dx$$

*) Untuk mempermudah perhitungan, gunakan konsep aljabar $\longrightarrow a^2 - b^2 = (a-b)(a+b)$

$$V_{cincin} = \int_{-1}^{2} \pi \left[(x+3+x^2+1)(x+3-x^2-1) \right] dx$$
$$= \int_{-1}^{2} \pi \left[(x^2+x+4)(-x^2+x+2) \right] dx$$
$$= \int_{-1}^{2} \pi \left[-x^4 + \cancel{x^3} + 2x^2 - \cancel{x^3} + x^2 + 2x - 4x^2 + 4x + 8 \right] dx$$
$$= \int_{-1}^{2} \pi \left[-x^4 - x^2 + 6x + 8 \right] dx$$
$$= \pi \int_{-1}^{2} -x^4 - x^2 + 6x + 8\, dx$$
$$= \pi \left[-\frac{1}{5}x^5 - \frac{1}{3}x^3 + 3x^2 + 8x \right]_{-1}^{2}$$
$$= \pi \left[-\frac{32}{5} - \frac{8}{3} + 12 + 16 - \left(\frac{1}{5} + \frac{1}{3} + 3 - 8 \right) \right]$$
$$= \pi \left[-\frac{32-1}{5} - \frac{8-1}{3} + 28 + 5 \right]$$
$$= \pi \left[-\frac{33}{5} - 3 + 33 \right]$$
$$= \pi \left[30 - \frac{33}{5} \right] \longrightarrow \frac{117}{5}\pi$$

$$\boxed{\text{Jawab} = \frac{117}{5}\pi}$$

Latihan Soal :

Carilah volume benda putar thd sumbu-x

a). $y = x^3+1$; $x=1$; $x=2$; $y=0$

b). $y = 9-x^2$, $y=0$

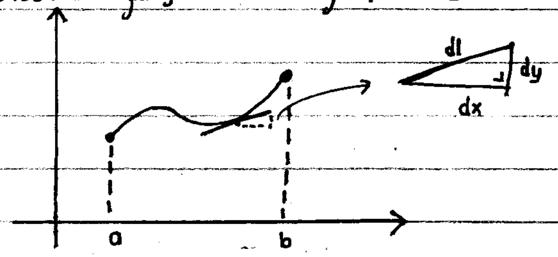
Carilah volume benda putar thd. sumbu-y

a). $x = y^2$; $x = y+2$

b). $x = \sqrt{9-y^2}$, $y=1$, $y=3$, $x=0$

4.3 Panjang Suatu Kurva

Diberikan fungsi kontinu f pada $[a, b]$



$$dl^2 = dx^2 + dy^2$$

$$dl = \sqrt{dx^2 + dy^2}$$

* Faktorkan dx^2 sehingga diperoleh :

$$dl = \sqrt{dx^2\left[1 + \frac{dy^2}{dx^2}\right]}$$

$$= dx\sqrt{1 + [f'(x)]^2}$$

$$\cancel{dl = \sqrt{1 + [f'(x)]^2}}$$

Rumus Panjang Suatu kurva :

$$\boxed{S = \int_a^b \sqrt{1 + [f'(x)]^2}\, dx}$$

Contoh Soal :

Dapatkan panjang busur kurva $y = \frac{x^4}{16} + \frac{1}{2x^2}$ dari $x = 2$ ke $x = 3$

Jawab :

① Tentukan $f'(x)$

$$f(x) = \frac{x^4}{16} + \frac{1}{2x^2}$$

$$f'(x) = \frac{4}{16}x^3 + \frac{1}{2}\cdot(-2)\cdot\frac{1}{x^3}$$

$$= \frac{1}{4}x^3 - \frac{1}{x^3}$$

② Dapatkan $[f'(x)]^2$

$$[f'(x)]^2 = \left(\frac{1}{4}x^3 - \frac{1}{x^3}\right)\left(\frac{1}{4}x^3 - \frac{1}{x^3}\right)$$

$$= \left(\frac{1}{16}x^6 - \frac{1}{4} - \frac{1}{4} + \frac{1}{x^6}\right]$$

$$= \frac{1}{16}x^6 - \frac{1}{2} + \frac{1}{x^6}$$

③ Substitusi ke Rumus S

$$S = \int_2^3 \sqrt{1 + \frac{1}{16}x^6 - \frac{1}{2} + \frac{1}{x^6}}\, dx$$

$$= \int_2^3 \sqrt{\frac{1}{16}x^6 + \frac{1}{x^6} + \frac{1}{2}}\, dx$$

* Ubah bentuk $\frac{1}{16}x^6 + \frac{1}{x^6} + \frac{1}{2}$ menjadi kuadrat sempurna

$$\left(\frac{x^3}{4} + \frac{1}{x^3}\right)^2 = \frac{x^6}{16} + \frac{1}{2} + \frac{1}{x^6}$$

Sehingga :

$$= \int_2^3 \sqrt{\left(\frac{x^3}{4} + \frac{1}{x^3}\right)^2}\, dx$$

$$= \int_2^3 \frac{x^3}{4} + \frac{1}{x^3}\, dx$$

$$= \frac{1}{4}\cdot\frac{1}{4}x^4 + \frac{1}{-2}x^{-2}\Big]_2^3$$

$$= \frac{1}{16}x^4 - \frac{1}{2x^2}\Big]_2^3$$

$$= \frac{1}{16}(3)^4 - \frac{1}{2(3)^2} - \left(\frac{1}{16}(2)^4 - \frac{1}{2(2)^2}\right)$$

$$= \frac{81}{16} - \frac{1}{18} - \frac{16}{16} + \frac{1}{8}$$

$$= \frac{65}{16} + \left[-\frac{4}{72} + \frac{9}{72}\right]$$

$$= \frac{65}{16} + \frac{5}{72}$$

$$= \frac{585 + 10}{144}$$

$$= \frac{595}{144}$$

Latihan Soal

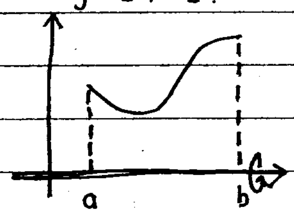
Dapatkan panjang busur kurva :

a. $y = 3x^{3/2} - 1$ dari $x = 0$ ke $x = 1$

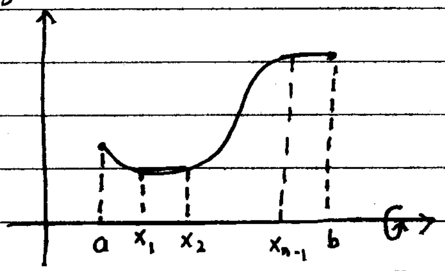
b. $x = \frac{1}{8}y^4 + \frac{1}{4}y^{-2}$ dari $y = 1$ ke $y = 4$

4.4 Luas Permukaan Benda Putar

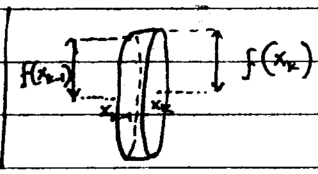
misal terdapat f fungsi kontinu tak negatif pada selang $[a, b]$.



Untuk memperoleh luas permukaan yang diputar terhadap sumbu-x, selang $[a,b]$ dibagi n sub-selang pada koordinat-x $\{a, x_1, x_2, \dots x_{n-1}, b\}$ dengan $\Delta x_1, \Delta x_2, \dots, \Delta x_n$ lebar subselang.



Diperoleh sebuah partisi :



Maka, diperoleh luas permukaannya adalah :

$$K = \sum_{k=1}^{n} 2\pi f(x_k)\, \Delta S_k$$

※ Catatan : Rumus Luas permukaan selimut tabung

$$\boxed{L = 2\pi r . t}$$

dalam hal ini : r = jari-jari = $f(x_k)$

t = tinggi = panjang busur

$$\Delta S_k = \sqrt{1 + [f'(x)]^2}\, \Delta x$$

ketika subselang diperbanyak hingga max $\Delta_k \to 0$, maka luas hampiran permukaan mendekati luas sebenarnya :

$$K = \int_a^b 2\pi f(x) \sqrt{1 + [f'(x)]^2}\, dx$$

Rumus Luas Permukaan Benda Putar

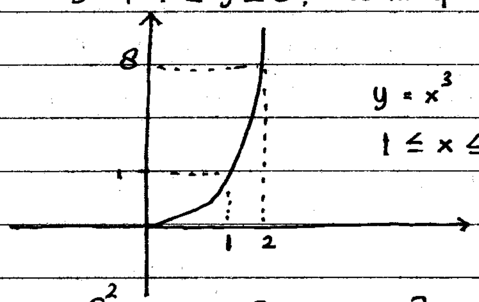
$$\boxed{K = \int_a^b 2\pi f(x) \sqrt{1 + [f'(x)]^2}\, dx}$$

apabila terhadap sb. y :

$$\boxed{K = \int_c^d 2\pi g(y) \sqrt{1 + [g'(y)]^2}\, dy}$$

Contoh Soal :

$x = \sqrt[3]{y}$, $1 \le y \le 8$, terhadap sumbu-x



$y = x^3$

$1 \le x \le 2$

$$K = \int_1^2 2\pi f(x) \left[\sqrt{1 + [f'(x)]^2}\right] dx$$

※ $f'(x) = 3x^2$

$$K = \int_1^2 2\pi . x^3 \sqrt{1 + (3x^2)^2}\, dx$$

$$= 2\pi \int_1^2 x^3 \sqrt{1 + 9x^4}\, dx$$

misal : $p = 1 + 9x^4$

$dp = 36x^3\, dx$

$x^3 dx = \dfrac{dp}{36}$

BA : $p = 1 + 9.16$

$= 145$

BB : $p = 1 + 9$

$= 10$

$$K = 2\pi \int_{10}^{145} \sqrt{p} \cdot \frac{dp}{36}$$

$$= \frac{\pi}{18} \cdot \left[\frac{2}{3} p\sqrt{p}\right]_{10}^{145}$$

$$= \frac{\pi}{27} \left[145\sqrt{145} - 10\sqrt{10}\right]$$

Latihan Soal !

1. $y = 7x$, $0 \le x \le 1$
2. $y = \sqrt{4 - x^2}$, $-1 \le x \le 1$

} terhadap sb. x

4.5 Titik Berat (Pusat Massa, Centroid)

Diberikan L bidang datar homogin yang dibatasi oleh kurva $y = f(x)$, sumbu-x, dan garis-garis $x=a$ dan $x=b$. Titik berat dari L adalah $Z(\bar{x}, \bar{y})$. dengan

$$\bar{x} = \frac{\int_a^b x \cdot y \, dx}{\int_a^b y \, dx} \qquad \bar{y} = \frac{\frac{1}{2}\int_a^b y^2 \, dx}{\int_a^b y \, dx}$$

Sementara itu, apabila daerah dibatasi oleh 2 buah kurva, maka persamaannya adalah :

$$\bar{x} = \frac{\int_a^b x(y_1 - y_2)\,dx}{\int_a^b (y_1 - y_2)\,dx} \qquad \bar{y} = \frac{\frac{1}{2}\int_a^b (y_1^2 - y_2^2)\,dx}{\int_a^b (y_1 - y_2)\,dx}$$

Dengan y_1 adalah kurva yang berada di atas kurva y_2

※ Penerapan Titik Berat untuk menentukan Volume Benda Putar dan Luas Permukaan Benda Putar

Dalil Guldin I : Volume Benda Putar

Jika suatu kurva datar diputar penuh thd. sumbu yang sebidang dengan luasan itu dan TIDAK memotong luasan itu, maka isi / volume benda putar sama dengan :

$$\boxed{V = 2\pi \cdot \bar{y}\, L}$$

dengan : L = luas dataran

$Z(\bar{x}, \bar{y})$ = titik berat L

V = Volume

$\bar{y}$ = jarak sumbu putar ke pusat massa

Dalil Guldin 2 : Luas Permukaan

Jika suatu busur dari suatu kurva datar diputar pada sumbu yang sebidang dengan busur itu dan TIDAK memotong busur itu, maka luas kulit benda putar :

$$\boxed{K = 2\pi\, \bar{y} \cdot S}$$

dengan : $Z(\bar{x}, \bar{y})$: titik berat busur AB

S : Panjang busur AB

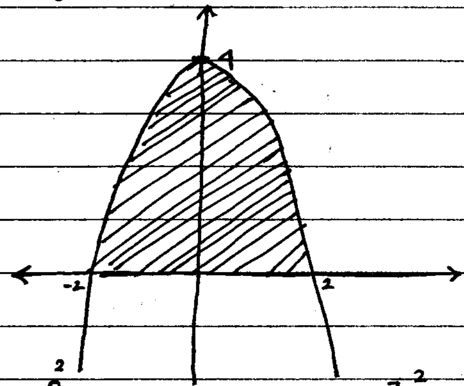
K : Luas Kulit

$\bar{y}$ = jarak sumbu putar ke pusat massa

Contoh Soal

Dapatkan titik berat bidang datar homogen yang dibatasi oleh :

$y = 4 - x^2$ dan sumbu-x



$$L = \int_{-2}^{2} 4 - x^2 \, dx = 4x - \frac{1}{3}x^3 \Big]_{-2}^{2}$$

$$= 4(2) - \frac{1}{3}(2)^3 - \left[4(-2) - \frac{1}{3}(-2)^3\right]$$

$$= 8 - \frac{8}{3} - \left(-8 + \frac{8}{3}\right)$$

$$= 8 - \frac{8}{3} + 8 - \frac{8}{3}$$

$$= 16 - \frac{16}{3}$$

$$= \frac{32}{3}$$

$$\bar{x} = \frac{\int_{-2}^{2} x \cdot (4 - x^2)\,dx}{\int_{-2}^{2} 4 - x^2 \, dx} = \frac{\int_{-2}^{2} 4x - x^3 \, dx}{32/3}$$

$$= \frac{3}{32}\left[2x^2 - \frac{1}{4}x^4\right]_{-2}^{2}$$

$$= \frac{3}{32}\left[2(2)^2 - \frac{1}{4}(2)^4 - \left(2(-2)^2 - \frac{1}{4}(-2)^4\right)\right]$$

$$= \frac{3}{32}\left[8 - 4 - (8 - 4)\right]$$

$= \frac{3}{32}[0]$

$= 0$

\# Catatan : Apabila suatu daerah atau luasan simetris terhadap suatu sumbu, maka :

- Simetri sb. x $\longrightarrow \bar{y} = 0$
- Simetri sb. y $\longrightarrow \bar{x} = 0$

$$\bar{y} = \frac{\frac{1}{2}\int_{-2}^{2} (4-x^2)^2\,dx}{\int_{-2}^{2} 4-x^2\,dx} = \frac{\frac{1}{2}\int_{-2}^{2} 16-8x^2+x^4\,dx}{32/3}$$

$$= \frac{3}{2.32}\int_{-2}^{2} \cancel{16-8x^2+x^4\,dx}$$

$$= \frac{3}{64}\left[\cancel{16x} - \frac{8}{3}x^3 + \frac{1}{5}x^5\right]_{-2}^{2}$$

$$= \frac{3}{64}\left[16(2) - \frac{8}{3}(2)^3 + \frac{1}{5}(2)^5 - \left(16(-2) - \frac{8}{3}(-2)^3 + \frac{1}{5}(-2)^5\right)\right]$$

$$= \frac{3}{64}\left[32 - \frac{64}{3} + \frac{32}{5} - \left(-32 + \frac{64}{3} - \frac{32}{5}\right)\right]$$

$$= \frac{3}{64}\left[64 - \frac{128}{3} + \frac{64}{5}\right]$$

$$= \frac{3}{64}\left[\frac{192-128}{3} + \frac{64}{5}\right]$$

$$= \frac{3}{64}\left[\frac{64}{3} + \frac{64}{5}\right]$$

$$= 1 + \frac{3}{5}$$

$$= \frac{8}{5}$$, jadi, $\bar{y} = \frac{8}{5} = 1\frac{3}{5} = 1,6$

Jawab : $(\bar{x}, \bar{y}) = (0; 1,6)$ //

Latihan Soal :

1. $y = 6 - 3x - x^2$ dan $y = 3 - x$
2. $y^2 = x$ dan $y = -\frac{1}{8}x^2$

▶ Contoh Soal Dalil Guldin I

Tentukan isi benda putar daerah yang dibatasi : $y = 6 - 3x - x^2$ dan $x + y - 3 = 0$ terhadap sumbu putar $y = 3 - x$

① Menentukan Gambar daerah

$y = 6 - 3x - x^2$

Puncak $\longrightarrow x = -\frac{b}{2a} = -\frac{3}{2}$

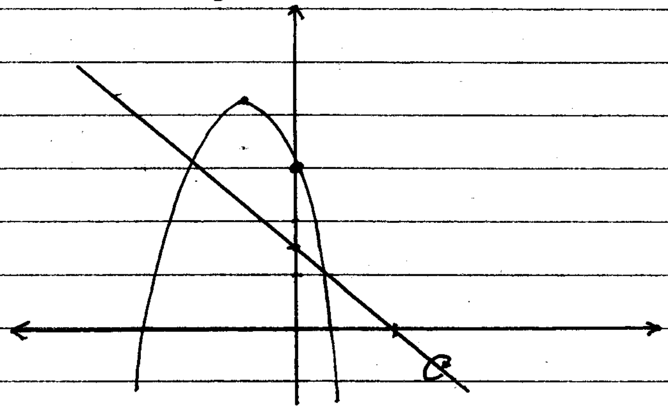
$$y = 6 - 3\left(-\frac{3}{2}\right) - \left(-\frac{3}{2}\right)^2$$

$$= 6 + \frac{9}{2} - \frac{9}{4}$$

$$= 6 + \frac{9}{4}$$

$$= \frac{33}{4}$$

Saat $x = 0 \longrightarrow y = 6$



$x + y - 3 = 0 \longrightarrow x = 0 ;\ y = 3$

$y = 0 ;\ x = 3$

② Menentukan Luas

Titik potong kurva :

$$y_1 = y_2$$

$$6 - 3x - x^2 = 3 - x$$

$$0 = x^2 + 2x - 3$$

$$(x+3)(x-1) = 0$$

$$x = -3 \vee x = 1$$

$$L = \int_{-3}^{1} (6 - 3x - x^2) - (3 - x)\,dx$$

$$= \int_{-3}^{1} 3 - 2x - x^2\,dx$$

$$= 3x - x^2 - \frac{1}{3}x^3\Big]_{-3}^{1}$$

$$= 3(1) - (1)^2 - \tfrac{1}{3}(1)^3 - \left[3(-3) - (-3)^2 - \tfrac{1}{3}(-3)^3\right]$$
$$= 3 - 1 - \tfrac{1}{3} - [-9 - 9 + 9]$$
$$= 11 - \tfrac{1}{3}$$
$$= \frac{32}{3}$$

③ Menentukan titik Pusat

$$\bar{x} = \frac{\int_{-3}^{1} x\,(6-3x-x^2-(3-x))\,dx}{\int_{-3}^{1} (6-3x-x^2-(3-x))\,dx}$$
$$= \frac{\int_{-3}^{1} x\,[3-2x-x^2]\,dx}{\frac{32}{3}}$$
$$= \frac{3}{32}\int_{-3}^{1} 3x - 2x^2 - x^3\,dx$$
$$= \frac{3}{32}\left[\frac{3}{2}x^2 - \frac{2}{3}x^3 - \frac{1}{4}x^4\right]_{-3}^{1}$$
$$= \frac{3}{32}\left[\frac{3}{2} - \frac{2}{3} - \frac{1}{4} - \left(\frac{3}{2}(-3)^2 - \frac{2}{3}(-3)^3 - \frac{1}{4}(-3)^4\right)\right]$$
$$= \frac{3}{32}\left[\frac{3}{2} - \frac{27}{2} - \frac{2}{3} - 18 - \frac{1}{4} + \frac{81}{4}\right]$$
$$= \frac{3}{32}\left[-\frac{24}{2} - \frac{56}{3} + \frac{80}{4}\right]$$
$$= \frac{3}{32}\left[-12 + 20 - \frac{56}{3}\right]$$
$$= \frac{3}{32}\left[8 - \frac{56}{3}\right]$$
$$= \frac{3}{32}\left[-\frac{32}{3}\right]$$
$$= -1$$

$$\bar{y} = \frac{\frac{1}{2}\int_{-3}^{1} (6-3x-x^2)^2 - (3-x)^2\,dx}{32/3}$$
$$= \frac{3}{64}\int_{-3}^{1} (6-3x-x^2-3+x)(6-3x-x^2+3-x)\,dx$$
$$= \frac{3}{64}\int_{-3}^{1} (3-2x-x^2)(9-4x-x^2)\,dx$$
$$= \frac{3}{64}\int_{-3}^{1} (27-12x-3x^2-18x+8x^2+2x^3-9x^2+4x^3+x^4)\,dx$$
$$= \frac{3}{64}\int_{-3}^{1} (27-30x-4x^2+6x^3+x^4)\,dx$$

$$\bar{y} = \frac{3}{64}\left[27x - 15x^2 - \frac{4}{3}x^3 + \frac{3}{2}x^4 + \frac{1}{5}x^5\right]_{-3}^{1}$$
$$= \frac{3}{64}\left[27 - 15 - \frac{4}{3} + \frac{3}{2} + \frac{1}{5} - \left(27(-3) - 15(-3)^2 - \frac{4}{3}(-3)^3 + \frac{3}{2}(-3)^4 + \frac{1}{5}(-3)^5\right)\right]$$
$$= \frac{3}{64}\left[12 + \frac{1}{6} + \frac{1}{5} - \left(-81 - 135 + 36 + \frac{243}{2} - \frac{243}{5}\right)\right]$$
$$= \frac{3}{64}\left[12 + 180 + \frac{1}{6} - \frac{243}{2} + \frac{1}{5} + \frac{243}{5}\right]$$
$$= \frac{3}{64}\left[192 - \frac{728}{6} + \frac{244}{5}\right]$$
$$= \frac{3}{64}\left[192 - \frac{364}{3} + \frac{244}{5}\right]$$
$$= \frac{3}{64}\left[\frac{2880 - 1820 + 732}{15}\right]$$
$$= \frac{\cancel{3}}{\cancel{64}}\left[\frac{\cancel{1792}^{\,28}}{\cancel{15}_{\,5}}\right]$$
$$= \frac{28}{5}$$
$$= 5\tfrac{3}{5}$$
$$= 5{,}6$$

④ Menentukan Centroid ke sumbu putar

$(\bar{x}, \bar{y}) = (-1 ; 5{,}6)$ ke $y = 3 - x \rightarrow x + y - 3 = 0$

$$d = \frac{ax + by - c}{\sqrt{a^2 + b^2}} = \frac{(-1) + (5{,}6) + 3}{\sqrt{2}}$$
$$= \frac{7{,}6}{\sqrt{2}}$$
$$= \frac{7{,}6}{2}\sqrt{2}$$
$$= 3{,}8\sqrt{2}$$

⑤ Menentukan Volume dengan Dalil Guldin I

$$V = 2\pi\, d \,.\, L$$
$$= 2\pi \,.\, 3{,}8\sqrt{2} \,.\, \frac{32}{3}$$
$$= 81{,}07\sqrt{2}\,\pi \text{ satuan Volume}$$

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